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Nonparametric Test for a class of Lifetime Distribution UBAC(2) Based on the Laplace Transform

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Abstract

Testing various classes of life distributions have been considered a very important problem in the literature during the last decades, and many authors tried to solve it. In this paper, a new statistic technique for testing exponentiality versus the class of life distribution used better than aged in increasing concave ordering (UBAC(2)) is introduced based on the Laplace transform. For this proposed test, the critical values, the pitman's asymptotic efficiency and the power of the test are calculated via simulation to assess the performance of the test. A new nonparametric test statistic for testing exponentiality versus UBAC(2) for right censored data is proposed, and the critical values are tabulated. Finally, real data are used for complete and censored data by using our proposed test.

Keywords: UBAC(2) class of life distribution, Testing of hypotheses, Asymptotic to normality, Pitman's efficiency, Monte Carlo method, The power of the test and censoring

Mathematics Subject Classification: 62

Introduction

Testing exponentiality versus some classes of life distributions forms the backbone in life testing, reliability theory, maintenance modeling and biological science. For more than four decades, many families of life distributions that characterize aging are declared, see [1, 2]. Many authors introduced many new classes and their corresponding test statistics for testing exponentiality against these classes.

The most well-known classes of life distributions can be mentioned such as: the increasing failure rate (IFR), decreasing mean residual life (DMRL), used better than age (UBA), used better than age in expectation (UBAE) and decreasing variance residual life (DVRL), see [3–6]. Testing exponentiality against the new better than used in the moment generating function order aging (NBU_{mgf}) class of life distribution based on Laplace transform has been given by Atallah et al. [7]. Abu-Youssef et al. [8] introduced a test statistic for used better than aged in convex tail ordering (UBACT) based on Laplace transform. Testing the new better than used in the increasing convex average (NBUCA) class of life distribution based on Laplace transform has been discussed by Al-Gashgari et al. [9]. Testing exponentiality against overall decreasing life in Laplace transform order has been studied by Mohamed et al. [10]. Nonparametric test using goodness of fit approach for testing exponentiality with scale parameter against the new better than used in the Laplace transform order (NBUL) class of life distribution has been introduced by Lotfy et al. [11]. Hassan [12] introduced a new test statistic for testing exponentiality against UBA class of life distributions based on Laplace transform. Non parametric testing for used better than aged in Laplace transform (UBAL) class of life distribution based on U-statistic is proposed by Abu-Youssef et al. [13].

Suppose a unit with life time X having a continuous life distribution $F(x)$, survival function $\overline{F}(x) = 1 - F(x)$ and finite mean $\mu = \int_0^\infty \overline{F}(x)\mathrm{d}x$.

Definition 1 The distribution function F is said to be UBAC(2) if

$$\int_t^\infty \overline{F}(u)\mathrm{d}u - \int_{x+t}^\infty \overline{F}(u)\mathrm{d}u \geq \left(1 - e^{-x}\right)\overline{F}(t), \tag{1}$$

which can be written as:

$$\nu(t) - \nu(x+t) \geq \left(1 - e^{-x}\right)\overline{F}(t), \tag{2}$$

see Ali [14].

This class includes many classes of life distributions. Willmot and Cai [15] showed that the UBA class includes the DMRL class. While Di Crescenzo [16] has shown that UBAE class is contained in the harmonic used better than aged in expectation (HUBAE) class of life distribution. And from Mohi el Din et al. [17], we have:

$$\text{IFR} \Rightarrow \text{UBA} \Rightarrow \text{UBAC(2)}$$

Thus we have

$$\begin{array}{lll} \text{IFR} \subset \text{DMRL} \subset & \text{UBA} \subset & \text{UBAC(2)} \\ & \cup & \\ & \text{UBAE} \subset & \text{HUBAE} \end{array}$$

For definition and properties of these classes, you can see Deshpande et al. [18] and Deshpande and Purohit [19].

The rest of this article is organized as follows: In "Methods" section, a new test statistic based on the Laplace transform for testing H_0: F is exponential against H_1: F is UBAC(2) class of life distribution and not exponential is studied. Pittman's asymptotic efficiency (PAE) of the test for several common distributions is discussed and the power of the test is estimated. Testing for censored data is proposed. In "Results and discussion" section, applications based on our proposed test for real data are discussed. Finally overall conclusion is given in "Conclusion" section.

Methods

Testing of hypotheses

In this section, a new test statistic based on Laplace transform for testing H_0:$\overline{F}$ is exponential against H_1:$\overline{F}$ is UBAC(2) and is not exponential is studied for a random sample $X_1, X_2, \ldots, X_n$ from a population with distribution function F is proposed.

We proposed the following measure of departure from H_0

$$\begin{aligned}\delta_{U_L} &= \int_0^\infty \int_0^\infty e^{-sx}\left[\nu(t) - \nu(x+t) - \overline{F}(t) + e^{-x}\overline{F}(t)\right]\mathrm{d}x\mathrm{d}t; \quad s > 0\\ &= \int_0^\infty \int_0^\infty e^{-sx}\nu(t)\mathrm{d}x\mathrm{d}t - \int_0^\infty \int_0^\infty e^{-sx}\nu(x+t)\mathrm{d}x\mathrm{d}t - \int_0^\infty \int_0^\infty e^{-sx}\overline{F}(t)\mathrm{d}x\mathrm{d}t\\ &\quad + \int_0^\infty \int_0^\infty e^{-(s+1)x}\overline{F}(t)\mathrm{d}x\mathrm{d}t\\ &= I_1 - I_2 - I_3 + I_4,\end{aligned} \tag{3}$$

where

$$\begin{aligned}I_1 &= \int_0^\infty \int_0^\infty e^{-sx}\nu(t)\mathrm{d}x\mathrm{d}t; \quad s > 0\\ &= \int_0^\infty e^{-sx}\mathrm{d}x \int_0^\infty \nu(t)\mathrm{d}t\\ &= \frac{\mu_{(2)}}{2s}.\end{aligned} \tag{4}$$

Let $x + t = u$, then

$$\begin{aligned}I_2 &= \int_0^\infty \int_0^\infty e^{-sx}\nu(x+t)\mathrm{d}x\mathrm{d}t; \quad s > 0\\ &= \int_0^\infty \int_0^\infty e^{-s(u-t)}\nu(u)\mathrm{d}u\mathrm{d}t\\ &= \int_0^\infty \int_0^\infty e^{-s(t-u)}\nu(t)\mathrm{d}u\mathrm{d}t\\ &= \frac{\mu_2}{2s} - \frac{\mu}{s^2} + \frac{1}{s^3}\left(1 - Ee^{-sX}\right),\end{aligned} \tag{5}$$

$$\begin{aligned}I_3 &= \int_0^\infty \int_0^\infty e^{-sx}\overline{F}(t)\mathrm{d}x\mathrm{d}t; \quad s > 0\\ &= \frac{\mu}{s},\end{aligned} \tag{6}$$

and

$$\begin{aligned} I_4 &= \int_0^\infty \int_0^\infty e^{-(s+1)x}\overline{F}(t)\mathrm{d}x\mathrm{d}t; \quad s>0 \\ &= \frac{\mu}{s+1}. \end{aligned} \tag{7}$$

Then

$$\delta_{U_\mathrm{L}} = \frac{s}{s^3(s+1)}\mu - \frac{1}{s^3}\left(1 - Ee^{-sX}\right); \quad s>0. \tag{8}$$

The empirical estimator $\hat{\delta}_{U_\mathrm{L}}$ of our test statistic can be given as follows:

$$\hat{\delta}_{U_\mathrm{L}} = \frac{1}{n}\sum_i \left[\frac{s}{s^3(s+1)}X_i - \frac{1}{s^3}\left(1 - e^{-sX_i}\right)\right]. \tag{9}$$

To make the test statistic scale invariant set

$$\hat{\Delta}_{U_\mathrm{L}} = \frac{\hat{\delta}_{U_\mathrm{L}}}{\overline{X}}. \tag{10}$$

Equation (9) can be rewritten as follows;

$$\hat{\delta}_{U_\mathrm{L}} = \frac{1}{n}\sum_i \phi(X_i) \tag{11}$$

where $\phi(X_i) = \frac{s}{s^3(s+1)}X_i - \frac{1}{s^3}\left(1 - e^{-sX_i}\right)$.

Set i = 1, then

$$\phi(X_1) = \frac{s}{s^3(s+1)}X_1 - \frac{1}{s^3}\left(1 - e^{-sX_1}\right). \tag{12}$$

Then $\hat{\delta}_{U_\mathrm{L}}$ is a classical U-statistics, see Lee [20].

Theorem 1 *As* $n \to \infty$, $\sqrt{n}\left(\hat{\delta}_{U_\mathrm{L}} - \delta_{U_\mathrm{L}}\right)/\sigma$ *is asymptotically normal with mean 0 and variance* $\sigma^2 = \mathrm{var}[\phi(X_1)]$, *where* $\phi(X_1)$ *is given in* (12).

Under H_0

$$\sigma^2(s) = \frac{2}{s^2(s+1)^3(2s+1)}. \tag{13}$$

Proof Using the theory of standard U-statistics and by direct calculations, we can find the mean equal 0 and the variance is given by

$$\sigma^2 = \mathrm{var}[\phi(X_1)],$$

Then

$$\sigma^2(s) = E\left[\frac{s}{s^3(s+1)}X_1 - \frac{1}{s^3}\left(1-e^{-sX_1}\right)\right]^2. \tag{14}$$

Under H_0:$E(\phi(X_1)) = 0$ and

$$\begin{aligned}\sigma_0^2(s) &= E[\phi(X_1)]^2 \\ &= \int_0^\infty \left[\frac{s}{s^3(s+1)}X_1 - \frac{1}{s^3}\left(1-e^{-sX_1}\right)\right]^2 e^{-x}\mathrm{d}x.\end{aligned} \tag{15}$$

Then (13) is given and the theorem is proved.

When $s = 1 \rightarrow \sigma_0^2(1) = \frac{1}{12}$

$$\hat{\Delta}_{U_\mathrm{L}}(1) = \frac{1}{n\overline{X}}\left[\frac{1}{2}X_1 - \left(1-e^{-X_1}\right)\right] \tag{16}$$

To use the above test, calculate $\sqrt{n}\hat{\Delta}_{\mathrm{U_L}}/\sigma_0$ and reject H_0 if this exceeds the normal variate value $Z_{1-\alpha}$. To illustrate the test, we calculate, by using Monte Carlo method, the empirical critical values of $\hat{\Delta}_{U_\mathrm{L}}$ in (16) for sample sizes 5(5)50. Table 1 gives the percentile points for 1%, 5%, 10%, 90%, 95% and 99%. The calculations are based on 10,000 simulated samples of sizes n = 5(5)80(10)100.

Pitman's asymptotic efficiency

In this section, we calculate PAE for UBAC(2) class of life distributions and compare our proposed test with tests of other well-known classes of life distributions on basis of PAE.

Here we choose K^* presented by Hollander and Prochan [21] for DMRL, $\hat{\delta}_2$ presented by Ahmad [5] for UBAE class, $\hat{\Delta}_{U_\mathrm{T}}$ presented by Abu-Youssef et al. [22] for UBACT class

Table 1 Critical values of $\hat{\Delta}_{U_\mathrm{L}}$

n	1%	5%	10%	90%	95%	99%
5	−0.295886	−0.207452	−0.160331	0.170163	0.217285	0.305718
10	−0.214839	−0.152308	−0.118988	0.114707	0.148027	0.210559
15	−0.174203	−0.123146	−0.0959403	0.0948708	0.122076	0.173133
20	−0.152911	−0.108695	−0.085134	0.0801133	0.103674	0.14789
25	−0.134738	−0.0951898	−0.0741165	0.0736852	0.0947585	0.134307
30	−0.128788	−0.0926854	−0.0734482	0.0614756	0.0807128	0.116815
35	−0.112569	−0.0791448	−0.0613346	0.0635806	0.0813908	0.114815
40	−0.108597	−0.0773309	−0.060671	0.0561765	0.0728364	0.104102
45	−0.0958797	−0.066402	−0.0506949	0.05947	0.0751771	0.104655
50	−0.0930731	−0.065108	−0.050207	0.0543046	0.0692056	0.0971706
55	−0.0917535	−0.06509	−0.0508824	0.0487655	0.0629731	0.0896367
60	−0.0866307	−0.0611023	−0.0474995	0.047906	0.0615088	0.0870372
65	−0.0865124	−0.0619855	−0.0489164	0.0427463	0.0558154	0.0803424
70	−0.0831569	−0.0595222	−0.0469285	0.0413999	0.0539936	0.0776284
75	−0.0819245	−0.0590911	−0.0469245	0.0384089	0.0505755	0.0734089
80	−0.0739367	−0.0518285	−0.0400481	0.0425755	0.0543558	0.0764641
90	−0.0696858	−0.0488419	−0.0377353	0.040163	0.0512696	0.0721135
100	−0.0648092	−0.0450349	−0.0344983	0.0394026	0.0499392	0.0697135

of life distribution based on U-Statistics and Λ_n introduced by Mahmoud et al. [10] for overall decreasing life (ODL) class of life distribution.

PAE of $\hat{\Delta}_{U_L}$ is given by:

$$\mathrm{PAE}\big(\Delta_{U_L}(\theta)\big) = \frac{1}{\sigma_0}\left|\frac{\mathrm{d}}{\mathrm{d}\theta}\Delta_{U_L}(\theta)|_{\theta\to\theta_0}\right|. \tag{17}$$

Two of the most commonly used alternatives (cf. Hollander and Proschan [23]) are:

(i) Linear failure rate family: $\overline{F}_\theta = e^{-x-\frac{\theta x^2}{2}}, \quad x > 0, \quad \theta > 0$

(ii) Makeham family: $\overline{F}_{2_\theta} = e^{-x-\theta(x+e^{-x}-1)}, \quad x > 0, \quad \theta > 0$

The null hypothesis is at $\theta=0$ for linear failure rate and Makeham families. The PAE's of these alternatives of our procedure are, respectively:

$$\mathrm{PAE}\big(\Delta_{U_L}, \mathrm{LFR}\big) = \left|-\frac{1}{\sigma_0}\left[\frac{s^2+s+1}{s^3(s+1)^3}\right]\right|, \quad s > 0 \tag{18}$$

$$\mathrm{PAE}\big(\Delta_{U_L}, \mathrm{Makeham}\big) = \left|\frac{1}{\sigma_0}\left[\frac{s}{s^3(s+1)^2(s+2)} - \frac{1}{s^3} - \frac{s}{2s^3(s+1)}\right]\right|, \quad s > 0 \tag{19}$$

From Table 2, our test statistic $\hat{\Delta}_{U_L}$ is more efficient than K^*, $\hat{\delta}_2$, $\hat{\Delta}_{U_T}$ and Λ_n for LFR and Makeham families.

Note that: Since $\hat{\Delta}_{U_L}$ defines a class with parameter s of test statistic, we choose s that maximizes the PAE of that alternatives. If we take $s=0.1$ then our test will have more efficiency than others.

Finally, the power of the test statistics $\hat{\Delta}_{U_L}$ is considered for 95% percentiles in Table 3 for three of the most commonly used alternatives [see Hollander and Proschan [23]], they are.

(i) Linear failure rate: $\overline{F}_\theta = e^{-x-\frac{\theta x^2}{2}}, \quad x > 0, \quad \theta > 0$

(ii) Makeham: $\overline{F}_\theta = e^{-x-\theta(x+e^{-x}-1)}, \quad x \geq 0, \quad \theta > 0$

(iii) Weibull: $\overline{F}_\theta = e^{-x^\theta}, \quad x \geq 0, \quad \theta > 0$

These distributions are reduced to exponential distribution for appropriate values of θ.

Testing for censored data

In this section, a test statistic is proposed to test H_0:$\overline{F}$ is exponential distribution with mean μ versus, H_1:$\overline{F}$ is UBAC(2) and not exponential distribution, with randomly right-censored data (RR-C)in many experiments. Censored data are usually the only

Table 2 PAE of Δ_{U_L}

Distribution	K^*	$\hat{\delta}_2$	$\hat{\Delta}_{U_T}$	Λ_n	$\hat{\Delta}_{U_L}$
F_1					
Linear failure Rate	0.81	0.63	0.748	0.982	1.41
F_2					
Makeham	0.29	0.385	0.248	0.218	0.55

Table 3 Power estimate of $\hat{\Delta}_{U_L}$

Distribution	θ	Sample size		
		$n=10$	$n=20$	$n=30$
F_{11}	2	0.9974	0.9994	0.9998
Linear failure	3	0.9977	0.9996	1
Rate	4	0.9977	0.9996	1
F_2	2	0.9966	0.9992	0.9997
Makeham	3	0.9973	0.9994	0.9998
	4	0.9974	0.9995	1
F_3	2	0.9971	0.9994	0.9998
Weibull	3	0.9986	0.9998	1
	4	0.9989	1	1

information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can be modeled as following:

Suppose n items are put on test, and $X_1, X_2, \ldots, X_n$ are independent and identically distributed (i.i.d) random variables according to a continuous life distribution F which denote their true life time. Let $Y_1, Y_2, \ldots, Y_n$ be (i.i.d) according to a continuous life distribution G and assume that X's and Y's are independent. In the randomly right-censored model, we observe the pairs (Z_i, δ_i), $i = 1, \ldots, n$, where $Z_i = \min(X_i, Y_i)$ and

$$\delta_i = \begin{cases} 1 \ \text{if}\, Z_i = X_i (i \text{ - th observation is uncensored}) \\ 0 \ \text{if}\, Z_i = Y_i (i \text{ - th observation is censored}) \end{cases}. \tag{20}$$

Let $Z_{(0)} < Z_{(1)} < \cdots < Z_{(n)}$ denoted the ordered of Z's and δ_i is the δ corresponding to $Z_{(i)}$, respectively. Using the Kaplan and Meier estimator [24] in the case of censored data (Z_i, δ_i), $i = 1, 2, \ldots, n$, then the proposed test statistic is given by (8) can be written using right censored data as

$$\hat{\delta}^c_{\mathrm{UL}} = \frac{s}{s^3(s+1)}\eta - \frac{1}{s^3}(1-\zeta), \tag{21}$$

where

$$\eta = \sum_{k=1}^{n}\left[\prod_{m=1}^{k-1} C_m^{\delta(m)}\left(Z_{(k)} - Z_{(k-1)}\right)\right],$$
$$\zeta = \sum_{j=1}^{n} e^{-sZ_{(j)}}\left[\prod_{p=1}^{j-2} C_p^{\delta(p)} - \prod_{p=1}^{j-1} C_p^{\delta(p)}\right]$$

and

$$\mathrm{d}F_n(Z_j) = \overline{F}_n(Z_{j-1}) - \overline{F}_n(Z_{j-2}),$$
$$c_k = \frac{n-k}{n-k+1}.$$

To make the test invariant, let

$$\hat{\Delta}^{c}_{\mathrm{UL}} = \frac{\hat{\delta}^{c}_{\mathrm{UL}}}{\overline{Z}}, \quad \overline{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i. \tag{22}$$

Table 4 shows the critical values percentiles of $\hat{\Delta}^{c}_{\mathrm{UL}}$ for sample size $n = 2(2)20(10)100$.

Results and discussion

Applications for complete data

Example 1 The following data represent 39 liver cancers patients taken from El Minia Cancer Center Ministry of Health Egypt Attia et al. [25]. The ordered life times (in days) are:

107, 18, 74, 20, 23, 20, 23, 24, 52, 105, 60, 31, 75, 107, 71, 107, 14, 49, 10, 15, 30, 26, 14, 87, 51, 17, 116, 67, 20, 14, 40, 14, 30, 96, 20, 20, 61, 150, 14.

Using Eq. (16), the value of test statistics, based on the above data is $\hat{\Delta}_{U_{\mathrm{L}}} = 0.504612$. The critical value at $\alpha = 0.05$ is 0.0792404, then we reject H_0 at $\alpha = 0.05$. Therefore, the data have UBAC(2) property.

Example 2 Consider the data in Abouammoh et al. [26]. These data represent set of 40 patients suffering from blood cancer (Leukemia) from one ministry of health hospital in Saudi Arabia. The ordered life times (in day) are:

0.315, 0.496, 0.699, 1.145, 1.208, 1.263, 1.414, 2.025, 2.036, 2.162, 2.211, 2.370, 2.532, 2.693, 2.805, 2.910, 2.912, 3.192, 3.263, 3.348, 3.348, 3.427, 3.499, 3.534, 3.718, 3.751,

Table 4 Critical values of $\hat{\Delta}^{c}_{U_{\mathrm{L}}}$

n	90%	95%	99%
2	0.929563	0.94527	0.974748
4	0.913392	0.924499	0.945343
6	0.906202	0.91527	0.932289
8	0.90189	0.909744	0.924482
10	0.898921	0.905945	0.919128
12	0.896698	0.903111	0.915145
14	0.894933	0.90087	0.912011
16	0.893459	0.899013	0.909435
18	0.892162	0.897398	0.907223
20	0.890929	0.895896	0.905217
30	0.889222	0.893278	0.900889
40	0.887317	0.890829	0.89742
50	0.888079	0.891221	0.897116
60	0.885845	0.888713	0.894095
70	0.88483	0.887485	0.892468
80	0.884101	0.886584	0.891245
90	0.883526	0.885868	0.890262
100	0.883054	0.885275	0.889444

3.858, 3.986, 4.049, 4.244, 4.323, 4.323, 4.381, 4.392, 4.397, 4.647, 4.753, 4.929, 4.973, 5.074.

The value of test statistics, based on the above data is $\hat{\Delta}_{U_L} = 0.349681$. The critical value at $\alpha = 0.05$ is 0.0737887. This value leads to the rejecting of H_0 at the significance level $\alpha = 0.05$. Therefore, the data have UBAC(2) property.

Example 3 In an experiment at Florida State University to study the effect of methyl mercury poisoning on the life lengths of fish goldfish were subjected to various dosages of methyl mercury (Kochar [27]). At one dosage level the ordered times to death in week are:

6, 6.143, 7.286, 8.714, 9.429, 9.857, 10.143, 11.571, 11.714, 11.714

The value of test statistics, based on the above data is $\hat{\Delta}_{U_L} = 0.5648$. the critical value at $\alpha = 0.05$ is 0.148027. Then H_0 at the significance level $\alpha = 0.05$ is rejected. Therefore, the data have UBAC(2) property.

Applications for censored data

Example 1 Consider the following data in Mahmoud and Abdul Alim [28] that represent 51 liver cancers patients taken from the El Minia Cancer Center Ministry of Health in Egypt. Of them 39 represent whole life times (non-censored data) and the others represent censored data. The ordered life times (in days) are:

(i) Non-censored data

$$10, 14, 14, 14, 14, 14, 15, 17, 18, 20, 20, 20, 20, 20, 23, 23, 24, 26, 30, 30,$$

$$31, 40, 49, 51, 52, 60, 61, 67, 71, 74, 75, 87, 96, 105, 107, 107, 107, 116, 150.$$

(ii) Censored data

$$30, 30, 30, 30, 30, 60, 150, 150, 150, 150, 150, 185.$$

It is found that the test statistic for the set of data $\hat{\Delta}^c_{\mathrm{UK}} = 1.02373$. The critical value is 0.890726, so we reject H_0 which states that the set of data have UBAC(2) property under significant level $\alpha = 0.5$.

Conclusion

In this paper, a new test statistic technique for testing exponentiality versus UBAC(2) class of life distribution based on Laplace transform is proposed. Pitman's asymptotic efficiencies of our proposed test are calculated for LFR and Makeham families. It is proved that our test is more efficient than other tests. Critical values of this test are tabulated for complete and censored data. Also the powers of this test are estimated for some famously alternative distributions in reliability such as LFR and Makeham. Finally, examples in different areas are used as practical applications of our proposed test.

Abbreviations
UBAC: Used better than aged in increasing concave ordering; IFR: Increasing failure rate; DMRL: Decreasing mean residual life; UBA: Used better than age; UBAE: Used better than age in expectation; DVRL: Decreasing variance residual life; UBACT: Used better than aged in convex ordering upper tail; NBUCA: New better than used in convex ordering; NBU_{mgf}: New better than used in the moment generating function order aging; UBAL: Used better than aged in Laplace transform; ODL: Overall decreasing life; NBUL: New better than used in Laplace transform; HUBAE: Harmonic used better than age in expectation; PAE: Pitman's asymptotic efficiency; LFR: Linear failure rate.

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